

## 12.9 ELECTRIC POTENTIAL OR RELATION BETWEEN ELECTRIC POTENTIAL AND ELECTRIC INTENSITY:

Electric potential is defined as:

*"The work done on unit positive charge in displacing it against the direction of electric field".*

Mathematically....

$$\text{Electric Potential} = \frac{\text{Work}}{\text{Charge}} \quad \text{----- (i)}$$

$$\text{OR} \quad V = \frac{W}{q_0}$$

### Explanation:

Let a very small test charge  $q_0$  be displaced from point "P" to point "Q" along an arbitrary path in the field as shown in the figure. The displacement of charge  $q_0$  is  $\Delta r$ . The force on this charge  $q_0$  in the field is given by:

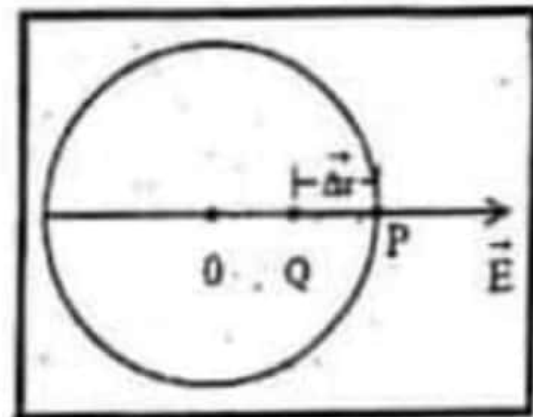
$$F = q_0 \vec{E}$$

$$\text{From } W = \vec{F} \cdot \vec{d}$$

$$\text{We have } W = q_0 \vec{E} \cdot \vec{\Delta r}$$

$$\text{OR } \frac{W}{q_0} = \vec{E} \cdot \vec{\Delta r}$$

$$\text{OR } V = \vec{E} \cdot \vec{\Delta r}$$



Thus, mathematically, electric potential is defined as, *"the dot product of electric field intensity and radial displacement vector of charge in the field"*

$$\text{OR } V = E \Delta r \cos \theta$$

Since displacement of charge is against the electric field, therefore,

$$\theta = 180^\circ$$

$$\text{and } V = E \Delta r \cos 180^\circ$$

$$\cos 180^\circ = -1$$

$$V = -E \Delta r$$

$$\text{OR } E = -\frac{V}{\Delta r}$$

This result shows that electric intensity is the negative potential gradient.

### S. I. Unit:

From eq.(1) 
$$\text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$$

**Definition of one Volt:**

*"If one coulomb charge is displaced against the field by doing one Joule work on it then the electric potential is said to be of one volt".* (2013)

Prove that  $\frac{1\text{Volt}}{\text{meter}} = \frac{1\text{Newton}}{\text{Coulomb}}$ , name the physical quantity which has these units.

L.H.S =  $\frac{1\text{Volt}}{\text{meter}}$   
 =  $\frac{\text{Joule/Coulomb}}{\text{meter}}$   $\therefore \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$   
 =  $\frac{\text{Joule}}{\text{Coulomb} \times \text{meter}}$   
 =  $\frac{\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}}$   $\therefore \text{Joule} = \text{Newton} \times \text{meter}$   
 $\frac{1\text{Volt}}{\text{meter}} = \frac{\text{Newton}}{\text{Coulomb}}$  Proved

OR

$\frac{1\text{Newton}}{\text{Coulomb}} = \frac{1\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}}$   
 =  $\frac{\text{Joule}}{\text{Coulomb} \times \text{meter}}$   $\therefore \text{Newton} \times \text{meter} = \text{Joule}$   
 =  $\frac{\text{Joule}}{\text{Coulomb}} \times \frac{1}{\text{meter}}$   
 =  $\text{Volt} \times \frac{1}{\text{meter}}$   $\therefore \frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$   
 $\frac{1\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{meter}}$  Proved the physical quantity is electric intensity.

**12.10 ELECTRIC POTENTIAL NEAR AN ISOLATED POINT CHARGE:**

OR

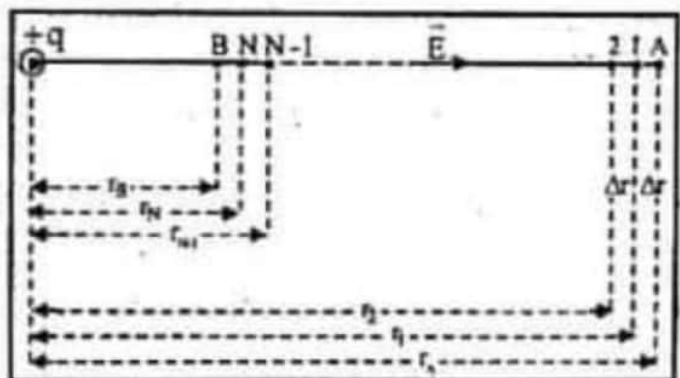
**ABSOLUTE ELECTRIC POTENTIAL**

**Definition:**

The absolute potential at a point is the work done in taking a unit positive charge from a point at infinity having zero potential to the point against the electric field intensity.

**Explanation:**

Consider two points A and B in a straight line at distances  $r_A$  and  $r_B$  respectively from an isolated point charge  $+q$  as shown in the figure. In order to determine the electric potential at B, a test charge  $q_0$  is moved from A to B. For this purpose some work has to be performed on  $q_0$ . Since electric intensity does not remain constant through A to B, so we divide the whole distance into a large number of small and equal



distances  $r_1, r_2, \dots, r_N$ , such that electric field which is the geometric mean over any surface assumed to be constant at all surfaces.

First, we determine the work done in moving charge  $q_0$  from point A to point 1

$$\begin{aligned} \Delta W &= Fd \cos\theta \\ \text{Here } F &= q_0 E \\ d &= \Delta r \\ \text{and } \theta &= 180^\circ \\ \therefore \Delta W_{A \rightarrow 1} &= q_0 E \Delta r \cos 180^\circ \\ \Delta W_{A \rightarrow 1} &= -q_0 E \Delta r \\ \text{OR } \frac{\Delta W_{A \rightarrow 1}}{q_0} &= -E \Delta r \\ \text{but } \frac{\Delta W_{A \rightarrow 1}}{q_0} &= \Delta V_{A \rightarrow 1} \\ \therefore \Delta V_{A \rightarrow 1} &= -E \Delta r \end{aligned}$$

Electric field intensity due to an isolated point charge is given by:

$$\begin{aligned} E &= \frac{Kq}{r^2} \\ \therefore \Delta V_{A \rightarrow 1} &= -Kq \frac{(r_A - r_1)}{r^2} \end{aligned}$$

• Where 'r' is the geometric mean distance of  $q_0$  when moved from point A to point 1.

$$\begin{aligned} r &= \sqrt{r_A r_1} \\ r^2 &= r_A r_1 \\ \therefore \Delta V_{A \rightarrow 1} &= -Kq \left( \frac{r_A - r_1}{r_A r_1} \right) \\ \Delta V_{A \rightarrow 1} &= -Kq \left( \frac{r_A}{r_A r_1} - \frac{r_1}{r_A r_1} \right) \\ \Delta V_{A \rightarrow 1} &= -Kq \left( \frac{1}{r_1} - \frac{1}{r_A} \right) \\ \text{Similarly } \Delta V_{1 \rightarrow 2} &= -Kq \left( \frac{1}{r_2} - \frac{1}{r_1} \right) \end{aligned}$$

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$$\Delta V_{N \rightarrow B} = -Kq \left( \frac{1}{r_B} - \frac{1}{r_N} \right)$$

Total electric potential from A to B is the algebraic sum of all electric potentials.

$$\begin{aligned} \Delta V_{A \rightarrow B} &= \Delta V_{A \rightarrow 1} + \Delta V_{1 \rightarrow 2} + \dots + \Delta V_{N \rightarrow B} \\ \Delta V_{A \rightarrow B} &= -Kq \left( \frac{1}{r_1} - \frac{1}{r_A} \right) - Kq \left( \frac{1}{r_2} - \frac{1}{r_1} \right) - \dots + Kq \left( \frac{1}{r_B} - \frac{1}{r_N} \right) \\ \Delta V_{A \rightarrow B} &= -Kq \left( \frac{1}{r_1} - \frac{1}{r_A} + \frac{1}{r_2} - \frac{1}{r_1} + \dots + \frac{1}{r_B} - \frac{1}{r_N} \right) \\ \Delta V_{A \rightarrow B} &= -Kq \left( -\frac{1}{r_A} + \frac{1}{r_B} \right) \\ \Delta V_{A \rightarrow B} &= -Kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right) \end{aligned}$$

Or potential difference  $V_B - V_A = -Kq \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$

The absolute potential at point B is obtained by considering the point A to be situated at infinity.

$$\Rightarrow \begin{aligned} r_A &= \infty \\ \frac{1}{r_A} &= \frac{1}{\infty} \\ \frac{1}{r_A} &= 0 \end{aligned}$$

$$\therefore V_B = -\frac{Kq}{r_B}$$

Absolute potential due to a point charge '+q' at a distance 'r' from it is given by:

$$V = -\frac{Kq}{r}$$

$$\text{OR } V = -\frac{1}{4\pi\epsilon_0} \frac{q}{r}$$