

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{r^2 + a^2 - 2ar - r^2 - y^2 + 2ar}{(r-a)^2 (r+a)^2} \right]$$

$$E = \frac{q}{4\pi\epsilon_0} \left[\frac{4ar}{(r^2 - a^2)^2} \right] \quad \text{As } p = q \times 2a$$

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{|\vec{p}| \cdot 2r}{(r^2 - a^2)^2} \quad \text{Required } \vec{E} \text{ acting along } \vec{BP}$$

If the dipole is short, i.e. $2a \ll r$ then

$$|\vec{E}| = \frac{|\vec{p}| \cdot 2r}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

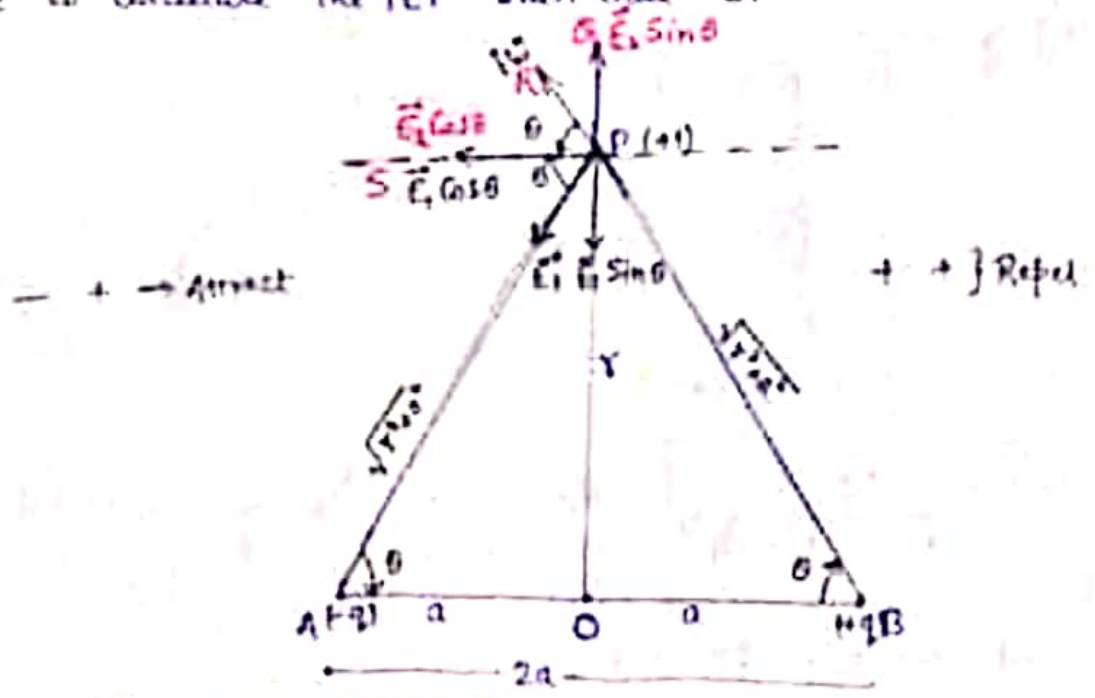
$$|\vec{E}| = \frac{2|\vec{p}|}{4\pi\epsilon_0 r^3}$$

Note :- $E \propto \frac{1}{r^3}$

ELECTRIC FIELD INTENSITY ON EQUATORIAL LINE OF DIPOLE.

Consider an electric dipole having charges $-q$ at A & $+q$ at B separated by a distance $2a$ with mid-point O.

Now let P be the point on Equatorial line of dipole where we have to calculate the $|\vec{E}|$ such that $OP = r$.



Also let $\angle PAO = \angle PBO = \theta$

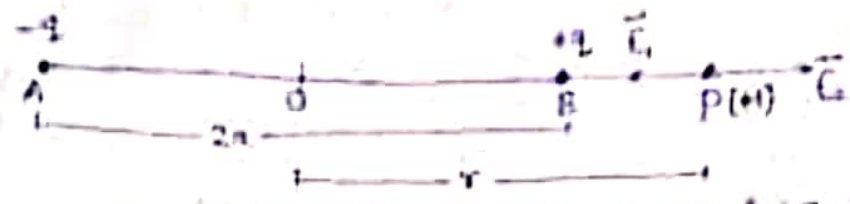
from figure $AP = BP = \sqrt{r^2 + a^2}$ (By pythagoras theorem)

Now, Electric field intensity \vec{E}_1 due to charge $-q$ at A is

[NOTE] IDEAL DIPOLE :- If the charges $\pm q$ becomes larger & the separation $2a$ becomes smaller & smaller, then the dipole is called as Ideal Dipole.

- Net Charge of an Ideal electric dipole is Zero.
- If centre of mass of +ve charge, coincides with centre of mass of -ve charge the molecule behaves non-polar as $(2a \rightarrow 0)$

6 a) ELECTRIC FIELD INTENSITY ON AXIAL LINE OF DIPOLE



Consider an electric dipole having charges $-q$ & $+q$ located at a distance $2a$ apart

Let P be the point on axial line of dipole where we have to calculate electric field intensity such that $OP=r$. Where O is the mid of dipole.

Hence if \vec{E}_1 is the electric field intensity at P due to $-q$ charge at A then

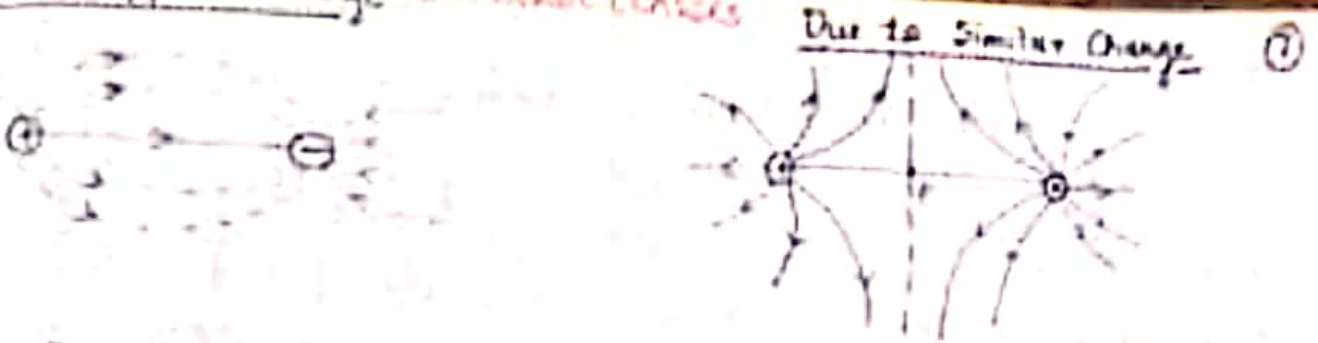
$$|\vec{E}_1| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{AP^2} \Rightarrow \boxed{\vec{E}_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r+a)^2}} \text{ Along } \vec{PA} \text{ --- (1)}$$

Again if \vec{E}_2 is the electric field intensity at P due to $+q$ charge at B then,

$$|\vec{E}_2| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{BP^2} \Rightarrow \boxed{\vec{E}_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2}} \text{ Along } \vec{BP} \text{ --- (2)}$$

As \vec{E}_1 & \vec{E}_2 are collinear vectors acting in opposite directions, since $|\vec{E}_2| > |\vec{E}_1|$. So resultant electric field intensity at P due to dipole is given by difference of two i.e.

$$\begin{aligned} |\vec{E}| &= |\vec{E}_2| - |\vec{E}_1| \\ |\vec{E}| &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r+a)^2} \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2 (r+a)^2} \right] \end{aligned}$$



Here P is the neutral point, where electric field is zero. Here P is the neutral point.

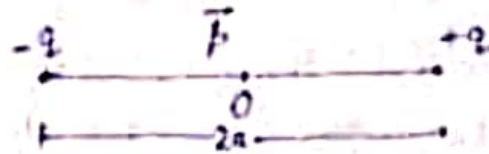
5 b) PROPERTIES OF ELECTRIC FIELD LINES.

- Electric field lines are discontinuous curves (they start from +ve charge & end at -ve charge).
- No Electric field lines exist inside the charged body.
- Tangent to the electric field lines at any pt gives the direction of \vec{E} at that pt.
- No two electric field lines of force can intersect each other.
- The electric field lines are always normal to the surface of conductor.
- The electric field lines contract longitudinally due to attraction b/w unlike charges.
- Electric field lines exert a lateral pressure due to repulsion of like charges.

6. ELECTRIC DIPOLE.

An electric dipole consists of a pair of equal & opposite pt charges separated by some small distance.

where $+q$ & $-q$ are equal & opposite charges separated by a distance $2a$, forms an Electric Dipole.



DIPOLE MOMENT

It is the measure of strengths of electric dipole. Defined as the product of magnitude of either charge and the distance b/w the charges. Denoted by \vec{P} , It is a Vector quantity, directed from -ve to +ve charge, given by,

$$\vec{P} = q \times (2a) \quad \text{Units are C-m.}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2} \hat{r} \text{ along } \vec{OP}$$

[NOTE] The Electric field vector (\vec{E}) around a +ve charge are directed radially outwards.

Electric field Intensity Due to Gp. of Charge.

\vec{E} due to gp of point charges is equal to vector sum of Electric field intensities due to individual charges at same pt. Same as Superposition principle,

i.e. $\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{Q_i}{r_i^2} \hat{r}_i \quad \text{--- (2)}$$

Where Q_1, Q_2, Q_3, \dots are the charges located at positions $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots$ from the test charge q_0 .

In terms of position Vectors.

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r_{12}^2} \hat{r}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r_{12}^3} \cdot \vec{r}_{12}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

5. ELECTRIC FIELD LINES.

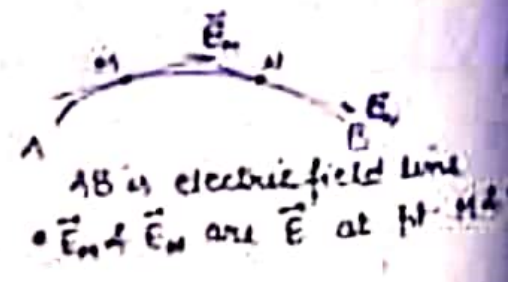
It is defined as a path either straight or curved such that tangent to it at any pt. gives the direction of Electric field Intensity at that point.



Electric field due to +ve charge.



Electric field due to -ve charge.



AB is electric field line
• \vec{E}_m & \vec{E}_n are \vec{E} at M & N