1. CAPACITORS

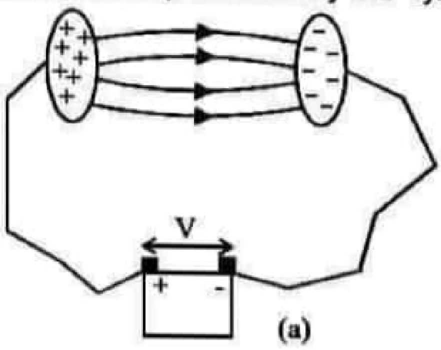
arry charges ±q and have a potential difference V, the capacitance of the capacitor is defined as the magnitude of the charge on one of the plates divided by the magnitude of the potential difference V between them

$$C = \frac{q}{V}$$

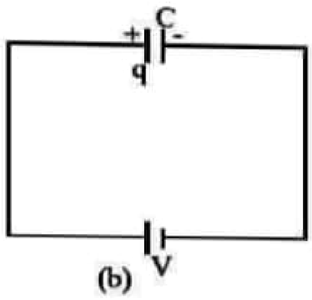
Capacitance depends on the size and shape of the plates and the material between them. It does not depend on q or V individually. The SI unit of capacitance is the farad (F)

1 farad = 1 coulomb/volt

In circuit, a capacitor is represented by the symbol ⊣⊢



(a)



(b)

2. TYPES OF CAPACITOR

(a) Parallel plate capacitor

(b) Spherical capacitor

(c) Cylindrical capacitor

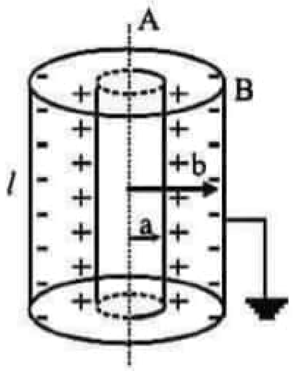
3. CAPACITY OF CAPACITOR - ITS CALCULATION

Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length l. If a charge q is given to the inner cylinder, induced charge -q will reach the inner surface of the outer cylinder. By symmetry, the electric field in the region between the cylinder is radially outward.

By Gauss's theorem, the electric field at a distance r from the axis of the cylinders is given by

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$

The potential difference between the cylinders is given by



$$V = \int_b^a \vec{E}.d\vec{r} = -\frac{1}{2\pi\varepsilon_0 l} q\int_b^a \frac{dr}{r}$$

$$= \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right)$$

$$q \qquad 2\pi\varepsilon_0 l$$

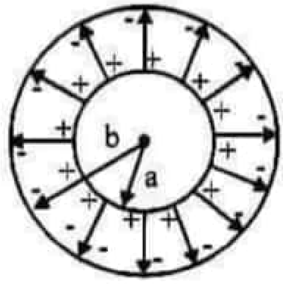
SPHERICAL CAPACITOR

A spherical capacitor consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential.

The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$



Hence, capacitance

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 ab}{b-a}$$

Illustration : 1 The plates of a parallel plate capacitor are 10mm apart and 1m² in area. The plates are in vacuum. A potential difference of 10,000 V is applied across a capacitor.

Calculate -

(a) the capacitance

(b) the charge on each plate

(c) the electric field in space between the plates

Solution : (a) $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times1}{10\times10^{-3}} = 8.85 \times 10^{-10}$ F = 0.0008 μF

(b) Q = CV = $(0.000885 \times 10^{-6}) \times (10{,}000)$ = 0.85 μC

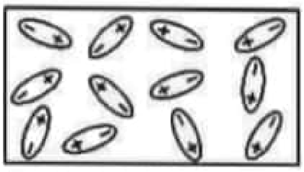
The plate at higher potential has a positive charge of +8.85 μC and the plate at lower potential has a negative charge of –8.85μC.

(c) $E = \frac{V}{d} = \frac{10{,}000}{0.01} = 1\times10^{6}\ V/m$

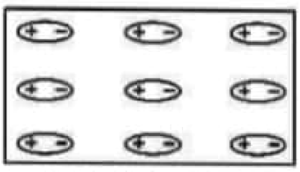
In monatomic materials, the centre of the negative charge coincides with the centre of the positive charge whereas in polyatomic materials, on the other hand, the center of the negative charge may or may not coincide with the centre of the positive charge distribution. If it does not coincide, each molecule behaves as a dipole with dipole moment $\vec{p}$. Such materials are known as polar materials.

If such a material is placed in an electric field, the individual dipoles experience torque due to the field and they try to align along the field.

The charge appearing on the surface of a dielectric when placed in an electric field is called ***induced charge***. As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called bound charge.



Dielectric in absence of electric field



Dielectric in presence of electric field

Because of the induced charges, an extra electric field is produced inside the material. If $\vec{E}_0$ be the applied field due to external sources and $\vec{E}_p$ be the field due to polarization. The resultant field is where K is a constant for given dielectric which has a value greater than one. This constant K is called the dielectric constant or relative permittivity of the dielectric.

Effect of dielectric on capacity of a capacitor

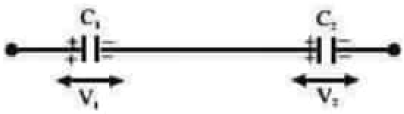
When certain non-conducting materials such a glass, paper or plastic are introduced, between the plates of a capacitor, its capacity increases. If capacity of a capacitor when completely filled with dielectric is C and that without dielectric is C_0

then $C = KC_0$

5. COMBINATION OF CAPACITORS

Series Combinations :

When two or more than two capacitors are connected in such a way that plates of capacitors are conneted with each other, the combination is known as series combination. [Only first plate of first capacitor and second plate of last capacitor is connected to source.]



When capacitors are connected in series, the magnitude of charge Q on each capacitor is same. The potential difference across C_1 and C_2 is different i.e., V_1 and V_2.

$$Q = C_1 V_1 = C_2 V_2$$

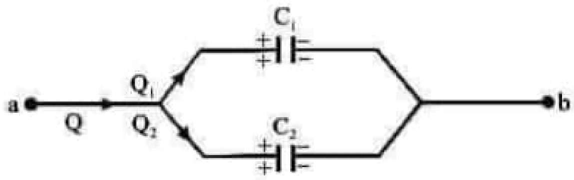
The total potential difference across combination is :

$$V = V_1 + V_2$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} \qquad \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

Parallel Combinations :

When two or more than two capacitors are connected in such a way that one plate of all capacitors are connected to one point and other plate of all capacitors are connected to other single point, such an arrangement of capacitors is known as parallel combination.



When capacitors are connected in parallel, the potential difference V across each is same and the charge on C_1, C_2 is different i.e., Q_1 and Q_2.

The total charge is Q given as :

$$Q = Q_1 + Q_2$$

$$Q = C_1V + C_2V$$

$$\frac{Q}{V} = C_1 + C_2$$

Equivalent capacitance between a and b is :

$$C = C_1 + C_2$$

The charges on capacitors is given as :

$$Q_1 = \frac{C_1}{C_1 + C_2}Q \quad \text{and} \quad Q_2 = \frac{C_2}{C_1 + C_2}Q$$

In case of more than two capacitors,

$$C = C_1 + C_2 + C_3 + C_4 + C_5 + \dots\dots\dots$$

Illustration : 2 Two capacitors of capacitance $C_1 = 6\ \mu F$ and $C_2 = 3\ \mu F$ are connected in series across a cell of emf 18 V.

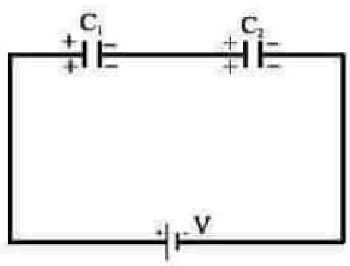
Calculate :

(a) the equivalent capacitance

(b) the potential difference across each capacitor

(c) the charge on each capacitor.

Solution : (a) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$



$$\Rightarrow C = \frac{C_1C_2}{C_1 + C_2} = \frac{6 \times 3}{6 + 3} = 2\mu F.$$

(b) $V_1 = \frac{C_2}{C_1 + C_2}V = \frac{3}{6+3} \times 18 = 6$ volts