12.9 ELECTRIC POTENTIAL OR RELATION BETWEEN ELECTRIC POTENTIAL AND ELECTRIC INTENSITY:

Electric potential is defined as:

"The work done on unit positive charge in displacing it against the directi... electric field".

Mathematically....

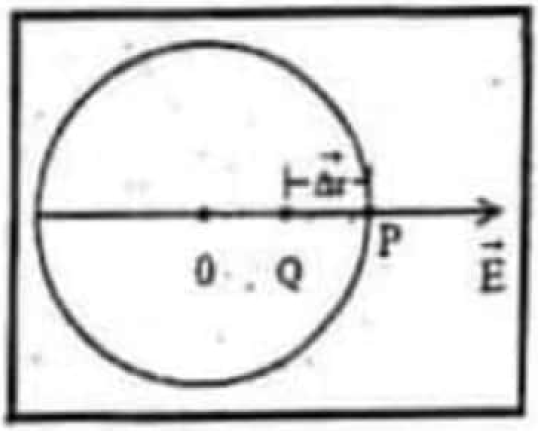
$$\text{Electric Potential} = \frac{\text{Work}}{\text{Charge}} \quad \text{———— (i)}$$

OR

$$V = \frac{W}{q_o}$$

Explanation:

Let a very small test charge q_0 be displaced from point "P" to point "Q" along... arbitrary path in the field as shown in the figure. The displacement of charge q_0 i... The force on this charge q_0 in the field is given by:



$$F = q_0 \vec{E}$$

From $W = \vec{F} . \vec{d}$

We have $W = q_0 \vec{E} . \overrightarrow{\Delta r}$

OR $\frac{W}{q_o} = \vec{E} . \overrightarrow{\Delta r}$

OR $V = \vec{E} . \overrightarrow{\Delta r}$

Thus, mathematically, electric potential is defined as, ***"the dot product of elec... field intensity and radial displacement vector of charge in the field"***

OR $V = E\Delta r \, Cos\theta$

Since displacement of charge is against the electric field, therefore,

$$\theta = 180^0$$

and $V = E\Delta r \, Cos180^0$

$$Cos180^0 = -1$$

$$V = -E\Delta r$$

OR $E = -\frac{V}{\Delta r}$

This result shows that electric intensity is the negative potential gradient.

S. I. Unit:

From eq.(1) $\quad \text{Volt} = \dfrac{\text{Joule}}{\text{Coulomb}}$

Definition of one Volt:

"If one coulomb charge is displaced against the field by doing one Joule work on it then the electric potential is said to be of one volt". **(2013)**

Prove that $\dfrac{1\text{Volt}}{\text{meter}} = \dfrac{1\text{Newton}}{\text{Coulomb}}$, name the physical quantity which has these units.

$$\text{L.H.S} = \frac{1\text{Volt}}{\text{meter}}$$

$$= \frac{\text{Joule/Coulomb}}{\text{meter}} \qquad \therefore \quad \text{Volt} = \frac{\text{Joule}}{\text{Coulomb}}$$

$$= \frac{\text{Joule}}{\text{Coulomb} \times \text{meter}}$$

$$= \frac{\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}} \qquad \therefore \quad \text{Joule} = \text{Newton} \times \text{meter}$$

$$\frac{1\text{Volt}}{\text{meter}} = \frac{\text{Newton}}{\text{Coulomb}} \quad \text{Proved}$$

OR

$$\frac{1\text{Newton}}{\text{Coulomb}} = \frac{1\text{Newton} \times \text{meter}}{\text{Coulomb} \times \text{meter}}$$

$$= \frac{\text{Joule}}{\text{Coulomb} \times \text{meter}} \qquad \therefore \quad \text{Newton} \times \text{meter} = \text{Joule}$$

$$= \frac{\text{Joule}}{\text{Coulomb}} \times \frac{1}{\text{meter}}$$

$$= \text{Volt} \times \frac{1}{\text{meter}} \qquad \therefore \quad \frac{\text{Joule}}{\text{Coulomb}} = \text{Volt}$$

$$\frac{1\text{Newton}}{\text{Coulomb}} = \frac{\text{Volt}}{\text{meter}}$$ Proved the physical quantity is electric intensity.

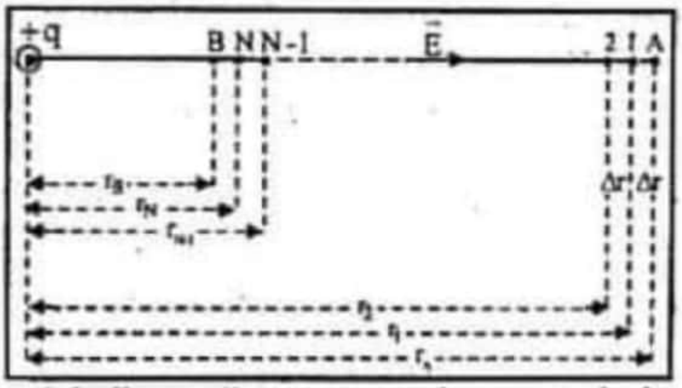
12.10 ELECTRIC POTENTIAL NEAR AN ISOLATED POINT CHARGE:

OR

ABSOLUTE ELECTRIC POTENTIAL

Definition:

The absolute potential at a point is the work done is taking a unit positive charge from a point at infinity having zero potential to the point against the electric field intensity.



Explanation:

Consider two points A and B in a straight line at distances r_A and r_B respectively from an isolated point charge +q as shown in the figure. In order to determine the electric potential at B, a test charge q_0 is moved from A to B. For this purpose some work has to be performed on q_0. Since electric intensity does not remain constant through A to B, so we divide the whole distance into a large number of small and equal distances $r_1, r_2,$ ------ r_N, such that electric field which is the geometric mean over any surface assumed to be constant at all surfaces.

First, we determine the work done in moving charge q_0 from point A to point 1

$$\Delta W = Fd\,Cos\theta$$

Here $F = q_0E$

$d = \Delta r$

and $\theta = 180^0$

$$\therefore \quad \Delta W_{A\to 1} = q_0E\Delta r\,Cos180^0$$

$$\Delta W_{A\to 1} = -\,q_0E\Delta r$$

OR

$$\frac{\Delta W_{A\to 1}}{q_0} = -\,E\Delta r$$

but

$$\frac{\Delta W_{A\to 1}}{q_0} = \Delta V_{A\to 1}$$

$$\therefore \quad \Delta V_{A\to 1} = -\,E\Delta r$$

Electric field intensity due to an isolated point charge is given by:

$$E = \frac{Kq}{r^2}$$

$$\therefore \quad \Delta V_{A\to 1} = -\,Kq\frac{(r_A - r_1)}{r^2}$$

Where 'r' is the geometric mean distance of q_0 when moved from point A to point 1.

$$r = \sqrt{r_A\,r_1}$$

$$r^2 = r_A r_1$$

$$\therefore \quad \Delta V_{A\to 1} = -\,Kq\left(\frac{r_A - r_1}{r_A\,r_1}\right)$$

$$\Delta V_{A\to 1} = -\,Kq\left(\frac{r_A}{r_A\,r_1} - \frac{r_1}{r_A\,r_1}\right)$$

$$\Delta V_{A\to 1} = -\,Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right)$$

Similarly

$$\Delta V_{1\to 2} = -\,Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$\Delta V_{N\to B} = -\,Kq\left(\frac{1}{r_B} - \frac{1}{r_N}\right)$$

Total electric potential from A to B is the algebraic sum of all electric potentials.

$$\Delta V_{A\to B} = \Delta V_{A\to 1} + \Delta V_{1\to 2} + \text{-------} + \Delta V_{N\to B}$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A}\right) - Kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \text{-----------} + Kq\left(\frac{1}{r_B} - \frac{1}{r_N}\right)$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_1} - \frac{1}{r_A} + \frac{1}{r_2} - \frac{1}{r_1} + \text{-----------} + \frac{1}{r_B} - \frac{1}{r_N}\right)$$

$$\Delta V_{A\to B} = -Kq\left(-\frac{1}{r_A} + \frac{1}{r_B}\right)$$

$$\Delta V_{A\to B} = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Or potential difference $V_B - V_A = -Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

The absolute potential at point B is obtained by considering the point A to be situated at infinity.

$$r_A = \infty$$

$$\Rightarrow \quad \frac{1}{r_A} = \frac{1}{\infty}$$

$$\frac{1}{r_A} = 0$$

$$\therefore \quad V_B = -\frac{Kq}{r_B};$$

Absolute potential due to a point charge '+q' at a distance 'r' from it is given by:

$$V = -\frac{Kq}{r}$$

OR

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$