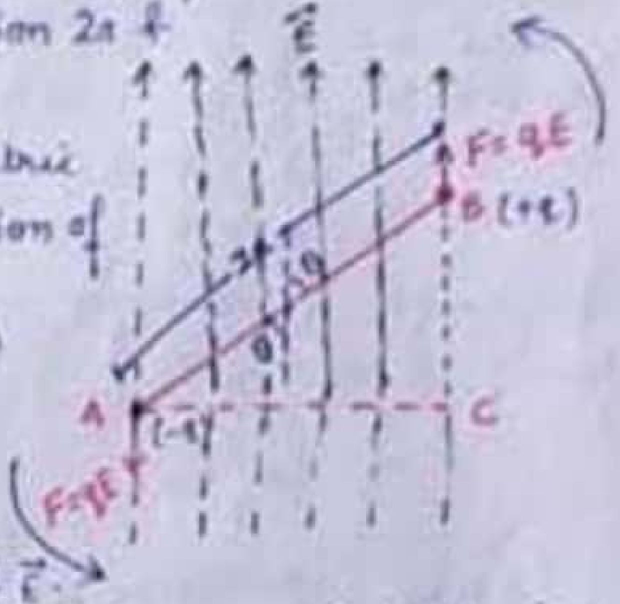
c) ELECTRIC DIPOLE IN A UNIFORM 2-DIMENSIONAL ELECTRIC-FIELD (Torque Acting On Uniform Electric field)

Consider an electric dipole having two charges -q at A & +q at B with separation 2a & mid-point O.

Let dipole be held in uniform electric field $\vec{E}$ at an angle θ with direction of $\vec{E}$.



Here the force on charge +q at B is

$\vec{F} = q\vec{E}$, acting along $\vec{E}$

Also, the force on charge -q at A is

$\vec{F} = q\vec{E}$, acting opposite of $\vec{E}$.

As these two forces are equal, opposite & parallel but having different line of action, so they form a couple which tends to rotate the dipole in the anti-clockwise direction.

Draw, $AC \perp \vec{E}$ i.e. AC represents arm of a couple.

Now, Torque = Force × Arm of Couple.

$$\tau = F \times AC$$
$$= F \times AB\sin\theta$$
$$= F \times 2a\sin\theta$$
$$\tau = q \cdot E\,(2a)\sin\theta \quad \{\vec{p} = 2a(q)\}$$
$$\tau = pE\sin\theta$$

In Vector form, $\boxed{\vec{\tau} = |\vec{p} \times \vec{E}|}$ where direct^n of $\vec{\tau}$ is given by right hand screw rule,

Note:- When $\vec{p}$ is along $\vec{E}$ i.e. θ=0 then τ=0.
then dipole is in stable equilibrium.

& When $\vec{p} \perp \vec{E}$ i.e. $\theta = 90^\circ$, then $\tau = p \cdot E$
then the torque will be maximum.

• The SI unit of τ is N·m & dimensional formula is $[ML^2T^{-2}]$

→ POTENTIAL ENERGY OF DIPOLE IN UNIFORM ELECTRIC FIELD

P.E of dipole is the energy possessed by the dipole by virtue of its particular position in the electric field.

Consider a dipole subjected to a external electricfield $\vec{E}$, so that torque acting on it is

$$\tau = PE\sin\theta \quad \text{---(1)}$$ where θ is the angle b/w $\vec{P}$ & $\vec{E}$

Here small amt of workdone in rotating the dipole by a small angle $d\theta$ against the torque τ is

$$dW = \tau \, d\theta$$
$$dW = PE\sin\theta \, d\theta$$

∴ Total amount of workdone in rotating the dipole from orientation (posit^n) θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} dW$$

i.e. $$W = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta \Rightarrow W = pE(-\cos\theta)_{\theta_1}^{\theta_2}$$

$$W = -pE(\cos\theta_2 - \cos\theta_1)$$

When the dipole is intially at right angle to $\vec{E}$ i.e. $\theta_1 = 90^\circ$ & we have to set $\theta_2 = \theta$, then

$$W = -pE(\cos\theta - \cos 90^\circ)$$
$$W = -pE\cos\theta$$

thus this workdone stored as Potential energy.

∴ $$PE = -pE\cos\theta$$

∴ In Vector form $$\boxed{U = -\vec{p} \cdot \vec{E}}$$

NOTE Potential Energy of dipole is a scalar quantity.
Its unit is Joule.