

$$\textcircled{3} \quad \tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$$

LHS.

$$\Rightarrow \tan^{-1} \frac{2}{11} + \frac{7}{24}$$

$$1 - \frac{2}{11} \times \frac{7}{24}$$

$$\Rightarrow \tan^{-1} \frac{2 \times 24 + 7 \times 11}{(11 \times 24) - 2 \times 7}$$

$$\Rightarrow \tan^{-1} \frac{48 + 77}{264 - 14}$$

$$\Rightarrow \tan^{-1} \left( \frac{125}{250} \right) \Rightarrow \tan^{-1} \frac{1}{2} \quad \text{Ans}$$

$$\textcircled{4} \quad 2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

$$\Rightarrow \tan^{-1} \frac{2 \times \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

$$\Rightarrow \tan^{-1} \frac{1}{1 - \frac{1}{4}} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

$$1+x^2 = \tan x / \cot x$$

$$x^2-1 = \operatorname{cosec} x / \sec x$$

$$1-x^2 = \sin / \cos$$

$$\Rightarrow \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

$$\Rightarrow \frac{\tan^{-1} \left( \frac{4}{3} + \frac{1}{7} \right)}{1 - \left( \frac{4}{3} \times \frac{1}{7} \right)} = \tan^{-1} \frac{31}{17}$$

$$\Rightarrow \frac{\tan^{-1} \left( \frac{28+3}{(7 \times 3)} \right)}{\left( \frac{(7 \times 3) - 4}{(7 \times 3)} \right)} = \tan^{-1} \frac{31}{17}$$

$$\Rightarrow \tan^{-1} \frac{31}{17} = \tan^{-1} \frac{31}{17}$$

LHS = RHS, verified!!

(5)  $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

$$x = \tan \theta \Rightarrow \theta = \tan^{-1} x$$

$$\Rightarrow \tan^{-1} \left( \frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\sqrt{\sec^2 \theta}-1}{\tan \theta} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\sec \theta - 1}{\tan \theta} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\frac{1}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\frac{1 - \cos \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \right) \Rightarrow \tan^{-1} \left( \frac{1 - \cos \theta}{\sin \theta} \right)$$

$$\Rightarrow \frac{\tan^{-1} 2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \Rightarrow \tan^{-1} \left( \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \right)$$

$$\Rightarrow \tan^{-1} \left( \tan \frac{\theta}{2} \right)$$

$$\Rightarrow \frac{\theta}{2} \Rightarrow \boxed{\frac{\tan^{-1} x}{2}} \text{ Ans}$$

⑥  $\tan^{-1} \frac{1}{\sqrt{x^2 - 1}}, |x| > 1$

$x = \operatorname{cosec} \theta$

$$\Rightarrow \tan^{-1} \frac{1}{\sqrt{\operatorname{cosec}^2 \theta - 1}}$$

$$\Rightarrow \tan^{-1} \frac{1}{\sqrt{\cot^2 \theta}}$$

$$\Rightarrow \tan^{-1} \frac{1}{\cot \theta}$$

$$\frac{\pi}{\pi} \cdot \frac{\pi}{\pi} \cdot \frac{\pi}{\pi} \cdot \frac{\pi}{\pi} \cdot \frac{\pi}{\pi}$$

$$\Rightarrow \tan^{-1} \tan \theta$$

$$\Rightarrow \theta \Rightarrow \underline{\tan^{-1} \operatorname{cosec}^{-1} x} \text{ Ans}$$

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$$\tan^{-1} \left( \sqrt{\frac{1 - \cos x}{1 + \cos x}} \right), \quad x < \pi$$

$$\Rightarrow \tan^{-1} \left( \sqrt{\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}} \right)$$

$$\Rightarrow \tan^{-1} \left( \sqrt{\frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}}} \right)$$

$$\Rightarrow \tan^{-1} \left( \sqrt{\tan^2 \frac{x}{2}} \right)$$

$$\Rightarrow \tan^{-1} \tan \frac{x}{2} \Rightarrow \frac{x}{2} \text{ Ans}$$

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$$\tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right), \quad x < \pi$$

2)

On dividing  $\cos x$ , we get

$$\tan^{-1} \left( \frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{1 - \tan x}{1 + \tan x} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\tan \frac{\pi}{4} - \tan x}{\tan \frac{\pi}{4} + \tan x \cdot \tan \frac{\pi}{4}} \right)$$

$$\Rightarrow x = \tan \frac{\pi}{4} \quad y = \tan x.$$

$$\Rightarrow \tan^{-1} \tan \frac{\pi}{4} - \tan^{-1} \tan x$$

$$\Rightarrow \boxed{\frac{\pi}{4} - x} \quad \text{Ans}$$

$$\textcircled{9} \quad \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}}, \quad |x| < a.$$

$$\Rightarrow x = a \sin \theta$$

$$\theta = \sin^{-1} \frac{x}{a}$$

$$\Rightarrow \tan^{-1} \left( \frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{a \sin \theta}{\sqrt{a^2 (1 - \sin^2 \theta)}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{a \sin \theta}{a \sqrt{\cos^2 \theta}} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\cancel{a} \sin \theta}{\cancel{a} \cos \theta} \right)$$

$$\Rightarrow \tan^{-1} \tan \theta$$

$$\Rightarrow \theta = \boxed{\sin^{-1} \frac{x}{a}} \quad \text{Ans}$$