ELECTRIC CHARGES AND FIELDS

Coulomb's Law: [Newtons N]

$F = k\frac{|q_1||q_2|}{r^2}$ where: F = force on one charge by the other [N]
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
q_1 = charge [C]

Electric Field inside a spherical shell: [N/C]

$E = \frac{kqr}{R^3}$
E = electric field [N/C]
q = charge [C]
r = distance from center of sphere to the charge [m]
R = radius of the sphere [m]

Electric Field outside a spherical shell: [N/C]

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where:
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$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
q_1 = charge [C]
q_2 = charge [C]
r = distance [m]

Electric Field: [Newtons/Coulomb or Volts/Meter]

$$E = k\frac{|q|}{r^2} = \frac{|F|}{|q|}$$

where:
E = electric field [N/C or V/m]
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
q = charge [C]
r = distance [m]
F = force

Electric field lines radiate outward from positive charges. The electric field is zero inside a conductor.

Relationship of k to ϵ_0:

$$k = \frac{1}{4\ \epsilon_0}$$

where:
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
ϵ_0 = permittivity of free space 8.85×10^{-12} [$C^2/N\,m^2$]

Electric Field due to an Infinite Line of Charge: [N/C]

$$E = \frac{\ }{2\ \epsilon_0 r} = \frac{2k\ }{r}$$

E = electric field [N/C]
= charge per unit length [C/m]
ϵ_0 = permittivity of free space 8.85×10^{-12} [$C^2/N\,m^2$]
r = distance [m]
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]

Electric Field due to ring of Charge: [N/C]

$$E = \frac{kqz}{(z^2 + R^2)^{3/2}}$$

or if z >> R, $E = \frac{kq}{z^2}$

E = electric field [N/C]
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
q = charge [C]
z = distance to the charge [m]
R = radius of the ring [m]

Electric Field due to a disk Charge: [N/C]

$$E = \frac{\ }{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

E = electric field [N/C]
= charge per unit area [C/m^2]
$\epsilon_0 = 8.85 \times 10^{-12}$ [$C^2/N\,m^2$]
z = distance to charge [m]
R = radius of the ring [m]

Electric Field due to an infinite sheet: [N/C]

$$E = \frac{\ }{2\epsilon_0}$$

E = electric field [N/C]
= charge per unit area [C/m^2]
$\epsilon_0 = 8.85 \times 10^{-12}$ [$C^2/N\,m^2$]

Electric Field inside a spherical shell: [N/C]

$$E = \frac{kqr}{R^3}$$

E = electric field [N/C]
q = charge [C]
r = distance from center of sphere to the charge [m]
R = radius of the sphere [m]

Electric Field outside a spherical shell: [N/C]

$$E = \frac{kq}{r^2}$$

E = electric field [N/C]
q = charge [C]
r = distance from center of sphere to the charge [m]

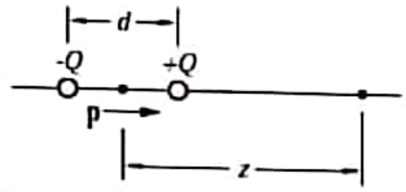
Average Power per unit area of an electric c magnetic field:

$$W/m^2 = \frac{E_m^2}{2\mu_0 c} = \frac{B_m^2 c}{2\mu_0}$$

W = watts
E_m = max. electric field [N/C]
$_0 = 4\pi \times 10^{-7}$
$c = 2.99792 \times 10^8$ [m/s]
B_m = max. magnetic field [T]

A positive charge moving in the same direction as the electri field direction loses potential energy since the potential c the electric field diminishes in this direction.
Equipotential lines cross EF lines at right angles.

Electric Dipole: Two charges of equal magnitude an opposite polarity separated by a distance d.



$$E = \frac{2kp}{z^3}$$

$$E = \frac{1}{2\ \epsilon_0}\frac{p}{z^3}$$

when $z \gg d$

E = electric field [N/C]
$k = 8.99 \times 10^9$ [$N\,m^2/C^2$]
ϵ_0 = permittivity of free space 8.85×10^{-12} $C^2/N\,m^2$
$p = qd$ [C m] "electric dipole moment" in the direction negative to positive
z = distance [m] from the dipole center to the point along the dipole axis where the electric field is to be measured

Deflection of a Particle in an Electric Field:

$$2ymv^2 = qEL^2$$

y = deflection [m]
m = mass of the particle [kg]
d = plate separation [m]
v = speed [m/s]
q = charge [C]
E = electric field [N/C or V/m
L = length of plates [m]

Potential Difference between two Points: [volts V]

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q} = -Ed$$

ΔPE = work to move a charge from A to B [N m or J]
q = charge [C]
V_B = potential at B [V]
V_A = potential at A [V]
E = electric field [N/C or V/m
d = plate separation [m]

Electric Potential due to a Point Charge: [volts V]

$$V = k\frac{q}{r}$$

V = potential [volts V]
k = 8.99 × 10^9 [N m²/C²]
q = charge [C]
r = distance [m]

Potential Energy of a Pair of Charges: [J, N m or C V]

$$PE = q_2 V_1 = k\frac{q_1 q_2}{r}$$

V_1 is the electric potential due to q_1 at a point P
$q_2 V_1$ is the work required to bring q_2 from infinity to point P

Work and Potential:

$$\Delta U = U_f - U_i = -W$$

$$U = -W_\infty$$

$$W = \mathbf{F} \cdot \mathbf{d} = Fd\cos$$

$$W = q\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

$$V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

U = electric potential energy [J]
W = work done on a particle by a field [J]
W = work done on a particle brought from infinity (zero potential) to its present location [J]
$\mathbf{F}$ = is the force vector [N]
$\mathbf{d}$ = is the distance vector over which the force is applied[m]
F = is the force scalar [N]
d = is the distance scalar [m]
= is the angle between the force and distance vectors
ds = differential displacement of the charge [m]
V = volts [V]
q = charge [C]

Flux: the rate of flow (of an electric field) [N m²/C]

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A}$$

$$= \int E(\cos\ )\, dA$$

Φ is the rate of flow of an electric field [N m²/C]
$\oint$ integral over a closed surface
$\mathbf{E}$ is the electric field vector [N/C]
$\mathbf{A}$ is the area vector [m²] pointing outward normal to the surface.

Gauss' Law:

$$\epsilon_0 \Phi = q_{enc}$$

$$\epsilon_0 \oint \mathbf{E} \cdot d\mathbf{A} = q_{enc}$$

ϵ_0 = 8.85 × 10^{-12} [C²/N m²]
Φ is the rate of flow of an electric field [N m²/C]
q_{enc} = charge within the gaussian surface [C]
$\oint$ integral over a closed surface
$\mathbf{E}$ is the electric field vector [J]
$\mathbf{A}$ is the area vector [m²] pointing outward normal to the surface.

CAPACITANCE

Parallel-Plate Capacitor:

$$C = \ \epsilon_0 \frac{A}{d}$$

C = capacitance [farads F]
κ = the dielectric constant (1)
ϵ_0 = permittivity of free space 8.85 × 10^{-12} C²/N m²
A = area of one plate [m²]
d = separation between plates [m]

Cylindrical Capacitor:

$$C = 2\ \ \epsilon_0 \frac{L}{\ln(b/a)}$$

C = capacitance [farads F]
κ = dielectric constant (1)
ϵ_0 = 8.85 × 10^{-12} C²/N m²
L = length [m]
b = radius of the outer conductor [m]
a = radius of the inner conductor [m]

Spherical Capacitor:

$$C = 4\ \ \epsilon_0 \frac{ab}{b-a}$$

C = capacitance [farads F]
κ = dielectric constant (1)
ϵ_0 = 8.85 × 10^{-12} C²/N m²
b = radius, outer conductor [m]
a = radius, inner conductor [m]

Maximum Charge on a Capacitor: [Coulombs C]

$$Q = VC$$

Q = Coulombs [C]
V = volts [V]
C = capacitance in farads [F]

For capacitors connected in series, the charge Q is equal for each capacitor as well as for the total equivalent. If the dielectric constant κ is changed, the capacitance is multiplied by κ, the voltage is divided by κ, and Q is unchanged. In a vacuum κ = 1, When dielectrics are used, replace ϵ_0 with $\kappa\epsilon_0$.

Electrical Energy Stored in a Capacitor: [Joules J]

$$U_E = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}$$

U = Potential Energy [J]
Q = Coulombs [C]
V = volts [V]
C = capacitance in farads [F]

Charge per unit Area: [C/m^2]

$= \frac{q}{A}$

= charge per unit area [C/m^2]
q = charge [C]
A = area [m^2]

Energy Density: (in a vacuum) [J/m^3]

$u = \frac{1}{2}\epsilon_0 E^2$

u = energy per unit volume [J/m^3]
ϵ_0 = permittivity of free space $8.85 \times 10^{-12}\ C^2/N\,m^2$
E = energy [J]

Capacitors in Series:

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \ldots$$

Capacitors in Parallel:

$$C_{eff} = C_1 + C_2 \ldots$$

Capacitors connected in series all have the same charge q. For parallel capacitors the total q is equal to the sum of the charge on each capacitor.

Time Constant: [seconds]

$\tau = RC$

= time it takes the capacitor to reach 63.2% of its maximum charge [seconds]
R = series resistance [ohms Ω]
C = capacitance [farads F]

Charge or Voltage after t Seconds: [coulombs C]

charging:

$$q = Q\left(1 - e^{-t/}\right)$$

$$V = V_S\left(1 - e^{-t/}\right)$$

discharging:

$$q = Qe^{-t/}$$

$$V = V_S\, e^{-t/}$$

q = charge after t seconds [coulombs C]
Q = maximum charge [coulombs C] $Q = CV$
e = natural log
t = time [seconds]
= time constant RC [seconds]
V = volts [V]
V_S = supply volts [V]

[Natural Log: when $e^b = x$, $\ln x = b$]

Drift Speed:

$$I = \frac{\Delta Q}{\Delta t} = \left(nqv_d A\right)$$

ΔQ = # of carriers × charge/carrier
Δt = time in seconds
n = # of carriers
q = charge on each carrier
v_d = drift speed in meters/second
A = cross-sectional area in meters^2

RESISTANCE

Emf: A voltage source which can provide continuous current [volts]

$\varepsilon = IR + Ir$

ε = emf open-circuit voltage of the battery
I = current [amps]
R = load resistance [ohms]
r = internal battery resistance [ohms]

Resistivity: [Ohm Meters]

$= \frac{E}{J}$

$= \frac{RA}{L}$

= resistivity [· m]
E = electric field [N/C]
J = current density [A/m^2]
R = resistance [ohms]
A = area [m^2]
L = length of conductor [m]

Variation of Resistance with Temperature:

$- {}_0 = {}_0 \ (T - T_0)$

= resistivity [· m]
= reference resistivity [· m]
= temperature coefficient of resistivity [K^{-1}]
T_0 = reference temperature
$T - T_0$ = temperature difference [K or °C]

CURRENT

Current Density: [A/m^2]

$I = \int \mathbf{J} \cdot d\mathbf{A}$

if current is uniform and parallel to $d\mathbf{A}$, then: $I = JA$

$J = (ne)V_d$

I = current [A]
J = current density [A/m^2]
A = area [m^2]
L = length of conductor [m]
e = charge per carrier
ne = carrier charge density [C/m^3]
V_d = drift speed [m/s]

Rate of Change of Chemical Energy in a Battery:

$P = I$

P = power [W]
I = current [A]
= emf potential [V]

Kirchhoff's Rules

1. The sum of the currents entering a junctions is equal to the sum of the currents leaving the junction.
2. The sum of the potential differences across all the elements around a closed loop must be zero.

Evaluating Circuits Using Kirchhoff's Rules

1. Assign current variables and direction of flow to all branches of the circuit. If your choice of direction is incorrect, the result will be a negative number. Derive equation(s) for these currents based on the rule that currents entering a junction equal currents exiting the junction.
2. Apply Kirchhoff's loop rule in creating equations for different current paths in the circuit. For a current path beginning and ending at the same point, the sum of voltage drops/gains is zero. When evaluating a loop in the direction of current flow, resistances will cause drops (negatives); voltage sources will cause rises (positives) provided they are crossed negative to positive—otherwise they will be drops as well.
3. The number of equations should equal the number of variables. Solve the equations simultaneously.