EX :- 2.2

Properties of Inverse trigonometry functions

(1) i) $\sin^{-1}\frac{1}{x} = \operatorname{cosec}^{-1}x$, $x \geq 1$ & $x \leq -1$

ii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x$, $x \geq 1$ & $x \leq -1$.

iii) $\tan^{-1}\frac{1}{x} = \cot^{-1}x$, $x > 0$

(2) i) $\sin^{-1}(-x) = -\sin^{-1}x$, $x \in [-1, 1]$

ii) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$

iii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$, $|x| \geq 1$

(3)

i) $\cos^{-1}(-x) = \pi - \cos^{-1}x$, $x \in [-1, 1]$

ii) $\sec^{-1}(-x) = \pi - \sec^{-1}x$, $|x| \geq 1$

(4) i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$, $x \in [-1, 1]$

ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in R$

iii) $\operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$, $|x| \geq 1$

(5) i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ $xy < 1$

ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$ $xy < -1$

iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $|x| < 1$

⑥ i) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$, $|x| \leq 1$

ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}$, $x \geq 0$

iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$, $-1 < x < 1$.

⑦ $3\sin\theta - 4\sin^3\theta = \sin 3\theta$

⑧ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

⑨ $\boxed{1-\cos\theta = 2\sin^2\frac{\theta}{2}}$

⑩ $\boxed{1+\cos\theta = 2\cos^2\theta/2}$

⑪ $\sin\theta = 2\sin\frac{\theta}{2}\cos\theta/2$

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(1)

$$\operatorname{cosec}^{-1}x = y.$$

$$x = \operatorname{cosec} y.$$

$$x = \frac{1}{\sin y}$$

$$\sin y = \frac{1}{x}$$

$$y = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\boxed{\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)}$$