Exercise - 2.2

① $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$, $x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$

⇒ RHS.

$\sin^{-1}(3x-4x^3)$

Let

$x = \sin\theta$

$\theta = \sin^{-1}x$

⇒ $\sin^{-1}(3\sin\theta - 4\sin^3\theta)$

⇒ $\sin^{-1}(\sin 3\theta)$

⇒ $3\theta \Rightarrow 3\sin^{-1}x$

LHS = RHS

verified!!

② $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$, $x \in \left[\frac{1}{2}, 1\right]$

⇒ RHS.

$\cos^{-1}(4x^3-3x)$

Let

$x = \cos\theta$

$\theta = \cos^{-1}x$

⇒ $\cos^{-1}(4\cos^3\theta - 3\cos\theta)$

⇒ $\cos^{-1}(\cos 3\theta)$

⇒ $3\theta \Rightarrow 3\cos^{-1}x$

LHS = RHS

verified!!

(3) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

LHS.

$\Rightarrow \tan^{-1}\dfrac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}$

$\Rightarrow \tan^{-1}\dfrac{\frac{2\times24+7\times11}{(11\times24)}}{\frac{(11\times24)-2\times7}{(11\times24)}}$

$\Rightarrow \tan^{-1}\dfrac{48+77}{264-14}$

$\Rightarrow \tan^{-1}\left(\frac{125}{250}\right) \Rightarrow \tan^{-1}\frac{1}{2}$ Ans

(4) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$\Rightarrow \tan^{-1}\dfrac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$\Rightarrow \tan^{-1}\dfrac{1}{1-\frac{1}{4}} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$1+x^2 = \tan x / \cot x$

$x^2-1 = \operatorname{cosec} x / \sec x$

$1-x^2 = \sin / \cos$.

$\Rightarrow \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

$\Rightarrow \tan^{-1}\frac{\left(\frac{4}{3}+\frac{1}{7}\right)}{1-\left(\frac{4}{3}\times\frac{1}{7}\right)} = \tan^{-1}\frac{31}{17}$

$\Rightarrow \tan^{-1}\left(\frac{28+3}{(7\times3)}\right) = \tan^{-1}\frac{31}{17}$

$\left(\frac{(7\times3)-4}{(7\times3)}\right)$

$\Rightarrow \tan^{-1}\frac{31}{17} = \tan^{-1}\frac{31}{17}$

LHS = RHS, verified !!

(5) $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$

$x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\Rightarrow \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right) \Rightarrow \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right)$

$\Rightarrow \tan^{-1}\dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \Rightarrow \tan^{-1}\left(\dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$

$\Rightarrow \tan^{-1}\left(\tan\dfrac{\theta}{2}\right)$

$\Rightarrow \dfrac{\theta}{2} \Rightarrow \boxed{\dfrac{\tan^{-1}x}{2}}$ Ans

(6) $\tan^{-1}\dfrac{1}{\sqrt{x^2-1}}$, $|x|>1$

$x = \operatorname{cosec}\theta$

$\Rightarrow \tan^{-1}\dfrac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$ | $\Rightarrow \tan^{-1}\dfrac{1}{\sqrt{\cot^2\theta}}$

$\Rightarrow \tan^{-1}\dfrac{1}{\cot\theta}$