Exercise 2(B)

1. Find the sum of the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ to n terms.
2. Find the sum of the series $1 + 2 + 4 + 8 + \ldots$ upto 12 terms.
3. Find the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ to n terms.
4. Find the sum of the series $4 + 44 + 444 + \ldots$ to n terms.
5. Find the sum of the series $0.5 + 0.55 + 0.555 + \ldots$ to n terms.
6. Find the sum of 8 terms of the G. P. 3, 6, 12, 24, ...
7. How many terms of the series $1 + 2 + 2^2 + \ldots$ must be taken to make 511.
8. If a be the first term, b the nth term and p the product of n terms of G. P., show that $p^2 = (ab)^n$.
9. The sum of n terms of a series is $a.2^n - b$, find its nth term. Are the terms of this series in G. P. ?
10. Find the nth term and the sum to n terms of the series :
    $1 + (1 + 2) + (1 + 2 + 2^2) + \ldots$
11. Find the sum of the series :
    $$1 + 3x + 9x^2 + 27x^3 + \ldots \text{ to } \infty.$$
12. Find the sum of the series :
    $$\frac{1}{5} + \frac{1}{7} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \text{ to } \infty$$
13. Find the sum of the geometric sequence $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots$ to 12th terms.
14. First term and the sum of an infinite G. P. are 2 and 3 respectively. Find the common ratio and sum upto n terms.
15. Find the sum upto infinite :
    $$\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5} + \frac{2}{5^6} + \ldots.$$
16. Express 0.5.4 as a rational number.
17. The sum of an infinite G. P. whose common ratio is numerically less than 1 is 32 and sum of the first two is 24, the find the G.P. (J. A. C., 2009)
18. The sum of infinite number of terms of a decreasing G. P. is 4 and sum of the squares of its terms to infintiy is $\frac{16}{3}$, find the G. P.
19. The first term of an infinite Geometric progression is 1 and the sum of the terms is 13, find the common ratio. (J. A. C., 2008, 12)
20. Find the sum to n terms : (J. A. C., 2007)
    $5 + 55 + 555 + \ldots\ldots$

Q1/2 Find the sum up to 30 terms

$1+2+4+8+16 \ldots$

Ans $a=1, \quad r=2, \quad n=30$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{1\left(1-(2)^{30}\right)}{1-2}$$

$$= \frac{1}{-1}\left[1-(2)^{30}\right]$$

$$= -1\left[1-(2)^{30}\right]$$

$$= \left[(2)^{30}-1\right]$$ Ans

Que How many terms of G.P 4, 2, 1 - - - - amount to $\frac{127}{16}$?

Ans $a = 4 = r = \frac{2}{4} = \frac{1}{2}$

$S_n = \frac{127}{16}$, $n = ?$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\frac{127}{16} = \frac{4\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}}$$

$$\frac{127}{16} = \frac{4\left[1-\left(\frac{1}{2}\right)^n\right]}{\frac{2-1}{2}}$$

$$\frac{127}{16} = \frac{4\times 2\left[1-\left(\frac{1}{2}\right)^n\right]}{1}$$

$$\frac{127}{16\times 8}\times 1 = \left[1-\left(\frac{1}{2}\right)^n\right]$$

or, $\frac{127}{128} - 1 = -\left(\frac{1}{2}\right)^n$

or, $\left(\frac{1}{2}\right)^n = 1 - \frac{127}{128} = \frac{128-127}{128}$

or, $\left(\frac{1}{2}\right)^n = \left(\frac{1}{128}\right) = \left(\frac{1}{2}\right)^7$

$\therefore n = 7$ Ans