(2.4) TO FIND THE SUM OF n TERMS OF A G. P.

For the G.P. let first term $= a$

and c.r. $= r$

and S_n denotes the sum of first n terms of G.P.

$$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-2} + ar^{n-1} \quad \ldots.(i)$$

Now multiplying both sides by r,

$$rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n \quad \ldots.(ii)$$

Subtracting eq. (ii) from (i),

$$S_n - rS_n = a - ar^n$$

or $$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r < 1 \qquad \textbf{Formula 1}$$

Multiplying the numerator and denominator by (–1),

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ when } r > 1 \qquad \textbf{Formula 3}$$

To find the sum of infinite terms of a G. P. when the value of the common ratio is less than one

For the G.P. let first term $= a$ and c.r. $= r$ where $|r| < 1$)

Let S_n denotes the sum of first n terms,

then $$S_n = a + ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-2} + ar^{n-1} \quad \ldots.(i)$$

$\therefore$ $$rS_n = ar + ar^2 + ar^3 + \ldots\ldots + ar^{n-1} + ar^n \quad \ldots.(ii)$$

Subtracting (ii) from (i),

$$S_n(1-r) = a - ar^n$$

or $$S_n = \frac{a - ar^n}{1-r}$$

or $$S_n = \frac{a}{1-r} - \frac{ar^n}{1-r} \quad \ldots.(iii)$$

$\because |r| < 1$ *i.e.*, $-1 < r < 1$.

$\therefore$ $n \to \infty$

$r^n \to 0$

$\therefore$ $\dfrac{ar^n}{1-r} \to 0$

So from (iii), $S_n = \dfrac{a}{1-r}$

SOLVED EXAMPLES

Example 1

Find the sum of 8 terms of the following G. P. :

$$\frac{1}{2} + 1 + 2 + \ldots\ldots\ldots$$

Solution :

The given series is

$$\frac{1}{2} + 1 + 2 + \ldots\ldots\ldots$$

Here first term $(a) = \dfrac{1}{2}$

and c.r. $(r) = 2$

$n = 8$

$$S_8 = \frac{a(r^8 - 1)}{r - 1} = \frac{\frac{1}{2}(2^8 - 1)}{2 - 1}$$

$$= \frac{1}{2}(256 - 1)$$

$$= \frac{255}{2} = 127\frac{1}{2}$$

Example 2

Find the sum of the following series :

$2 + 6 + 18 + \ldots\ldots\ldots + 4374.$

Solution :

The given series is in G.P.

Here First term $(a) = 2$

c.r. $(r) = 3\ (r > 1)$

Let $t_n = 4{,}374$

$\because$ $t_n = ar^{n-1}$

$4{,}374 = 2.(3)^{n-1}$

or $2{,}187 = 3^{n-1}$

or $3^7 = 3^{n-1}$

or $n - 1 = 7$

or $n = 8$

$\therefore \quad S_n = \dfrac{a(r^n - 1)}{r - 1}$

$= \dfrac{2(3^8 - 1)}{3 - 1} = \dfrac{2(3^8 - 1)}{2}$

$= 3^8 - 1 = 6{,}561 - 1 = 6{,}560.$ **Ans.**

Que Fourth term of G.P is 40 and tenth term is 2560 find the seventh term?

Ans $t_4 = 40$

$t_4 = ar^3 = 40$ ——— (i)

$t_{10} = ar^9 = 2560$ ——— (ii)

divide करने पर,

$$\frac{ar^9}{ar^3} = \frac{2560}{40}$$

$$r^6 = 64$$

$$r^6 = 2^6$$

$$\therefore r = 2$$

From eq (i)

$$t_4 = ar^3$$

$$40 = a \cdot (2)^3$$

$$a = \frac{40}{8} = 5$$

$$a = 5$$

$$t_7 = ar^6 = 5 \times (2)^6$$

$$= 5 \times 64$$

$$= 320 \text{ Ans}$$