3. Insert 4 H.Ms between 1 and $\frac{1}{16}$

$1, H_1, H_2, H_3, H_4, \ldots \frac{1}{16}$ .... are in H.P

$1, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \ldots 16$ .... are in A.P

$a = 1, \; n = 6, \; t_n = 16$

$$t_n = a + (n-1) \times d$$
$$16 = 1 + (6-1) \times d$$
$$16 = 1 + 5d$$
$$5d = 15$$
$$d = \frac{15}{5} = 3$$

$$\frac{1}{H_1} = 3 + 1 = 4$$

$$\frac{1}{H_2} = 4 + 3 = 7$$

$$\frac{1}{H_3} = 7 + 3 = 10$$

$$\frac{1}{H_4} = 10 + 3 = 13$$

$H.M_s = \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \frac{1}{13}$ Ans.

Q.6. The A.M between two numbers exceeds their G.M by 2 and the G.M. exceeds their H.M by $\frac{8}{5}$. Find the numbers.

Let two numbers is a and b

$$A = G + 2 \quad --- \langle i \rangle$$

$$G = H + \frac{8}{5}$$

$$H = G - \frac{8}{5} \quad --- \langle ii \rangle$$

$$G^2 = A \cdot H$$

$$G^2 = (G+2)\left(G - \frac{8}{5}\right)$$

$$\cancel{G^2} = \cancel{G^2} - \frac{8}{5}G + 2G - \frac{16}{5}$$

$$\cancel{5 = -8G + 10G + 6}$$

$$\frac{16}{\cancel{5}} = \frac{10G - 8G}{\cancel{5}}$$

$$2G = 16$$

$$G = 8 \quad ---- \langle iii \rangle$$

$$A = G + 2 = 8 + 2 = 10$$

$$A = 10$$

$$\frac{a+b}{2} = 10$$

$$b = 20 - a \quad --- \langle iv \rangle$$

$$G = 8$$

$$\sqrt{ab} = 8$$

$ab = (8)^2$

$ab = 64$

$a(20-a) = 64$

$20a - a^2 = 64$

$-a^2 + 20a - 64 = 0$

$-(a^2 - 20a + 64) = 0$

$a^2 - 20a + 64 = 0$

$a^2 - 16a - 4a + 64 = 0$

$a(a-16) - 4(a-16) = 0$

$a = 16, \; a = 4$

$(16, 4) \; (4, 16)$

Hence the two numbers is

$(16, 4) \; \{4, 16\}$ Ans.

Q.7. The Harmonic mean of two numbers is 4, their A.M. A and G.M. G. Satisfy the relation $2A + G^2 = 27$. Find the numbers

Let, two numbers is a and b

$2A + G^2 = 27$

or, $2A + A \cdot H = 27$

or, $2A + A \cdot 4 = 27$

or, $2A + 4A = 27$

or, $6A = 27$

or, $A = \cancel{\frac{27}{6}} \frac{9}{2}$

$\frac{a+b}{\cancel{2}} = \frac{9}{\cancel{2}}$ - - - <i>

$b = 9 - a$ - - <ii>

$$H.M = \frac{2ab}{a+b}$$

$$4 = \frac{2ab}{9}$$

$$2ab = 36$$

$$ab = 18$$

$$a(9-a) = 18$$

$$9a - a^2 - 18 = 0$$

$$a^2 - 9a + 18 = 0$$

$$a^2 - 6a - 3a + 18 = 0$$

$$a(a-6) - 3(a-6) = 0$$

$$a = 6, \quad a = 3$$

Hence the two numbers is {6, 3} or (3, 6)