

Exercise 3A Harmonic series

Q1. Find the sequence whose n^{th} term is $\frac{1}{3n-2}$. Is this sequence an H.P.

$\frac{1}{3n-2}$ are in H.P.

$3n-2$ are in A.P.

$a_n = 3n - 2$

$a_1 = 3 \times 1 - 2$ ($n=1$ 1st term)
 $= 3 - 2 = 1$

$a_2 = 3 \times 2 - 2$ ($n=2$ 2nd term)
 $= 6 - 2 = 4$

$a_3 = 3 \times 3 - 2$ ($n=3$ 3rd term)
 $= 9 - 2 = 7$

$a_4 = 3 \times 4 - 2$ ($n=4$ 4th term)
 $= 12 - 2 = 10$

1, 4, 7, 10, ... are in A.P.

$1, \frac{1}{4}, \frac{1}{7}, \frac{1}{10}, \dots$ are in H.P. Ans

Q2. Find the fourth term of the sequence $2, 2\frac{1}{2}, 3\frac{1}{3}, \dots$

$2, \frac{5}{2}, \frac{10}{3}, \dots$ are in H.P.

$\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \dots$ are in A.P.

$$a = \frac{1}{2}, \quad d = \frac{2}{5} - \frac{1}{2} = \frac{4-5}{10} = -\frac{1}{10}$$

$$t_n = a + (n-1)d$$

$$t_4 = \frac{1}{2} + (4-1) \times -\frac{1}{10}$$

$$= \frac{1}{2} - \frac{3}{10}$$

$$= \frac{5-3}{10} = \frac{2}{10} = \frac{1}{5}$$

$$\text{(Reciprocal)} t_4 = 5 \text{ Ans.}$$

Q.3. which term is $(-\frac{1}{26})$ of progression $\frac{1}{10}, \frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \dots$?

$$a = \frac{1}{10}$$

$\frac{1}{10}, \frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \dots$ are in H.P

10, 8, 6, 4, ... are in A.P

$$a = 10, d = -2$$

$$t_n = -26 \text{ (Reciprocal)}$$

$$t_n = a + (n-1)d$$

$$-26 = 10 + (n-1) \times -2$$

$$-26 = 10 - 2n + 2$$

$$2n = 12 + 26$$

$$n = \frac{38}{2} = 19$$

19th term Ans.

4. Find the seventh term of the series $\frac{1}{3} + \frac{8}{23} + \frac{9}{11} + \dots$

$$\frac{1}{3} + \frac{8}{23} + \frac{9}{11} + \dots \text{ are in H.P}$$

$$3 + \frac{23}{8} + \frac{11}{4} + \dots \text{ are in A.P}$$

$$u = 3, \quad d = \frac{23}{8} - 3 = -\frac{1}{8}$$

$$n = 7$$

$$a_7 = 3 + (7-1) \times -\frac{1}{8}$$

$$= 3 + \frac{1}{8} - \frac{7}{8}$$

$$= \frac{24 + 1 - 7}{8}$$

$$= \frac{18}{8}$$

$$= \frac{9}{4}$$

$$= \frac{9}{4}$$

Reciprocal

$$\left(\frac{9}{4}\right)^{-1} = \left(\frac{4}{9}\right) \text{ Ans.}$$